

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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**Pearson Edexcel International GCSE****Wednesday 7 June 2023**

Morning (Time: 2 hours)

Paper  
reference**4MA1/2HR****Mathematics A****PAPER 2HR****Higher Tier**

**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

**Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
- Anything you write on the formulae page will gain **NO** credit.

**Information**

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

**Advice**

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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## International GCSE Mathematics

## Formulae sheet – Higher Tier

**Arithmetic series**

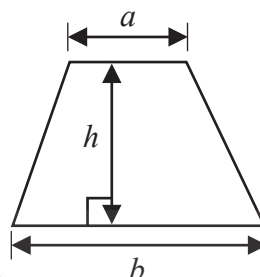
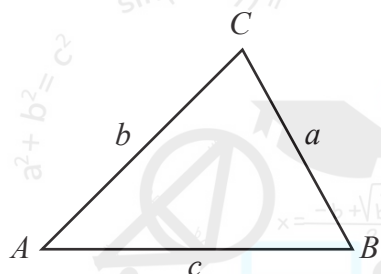
Sum to  $n$  terms,  $S_n = \frac{n}{2} [2a + (n-1)d]$

**The quadratic equation**

The solutions of  $ax^2 + bx + c = 0$  where  $a \neq 0$  are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Area of trapezium** =  $\frac{1}{2}(a+b)h$

**Trigonometry****In any triangle ABC**

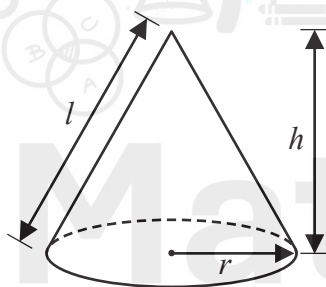
**Sine Rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

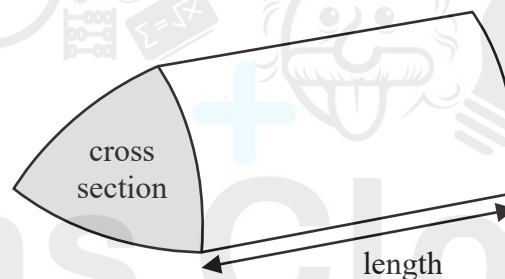
**Area of triangle** =  $\frac{1}{2}ab \sin C$

**Volume of cone** =  $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** =  $\pi r l$

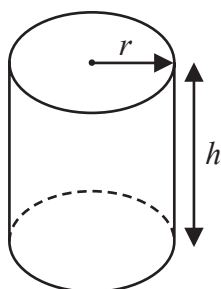
**Volume of prism**

= area of cross section  $\times$  length



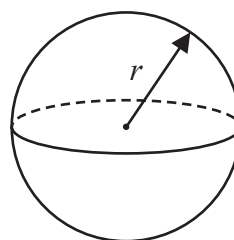
**Volume of cylinder** =  $\pi r^2 h$

**Curved surface area of cylinder** =  $2\pi r h$



**Volume of sphere** =  $\frac{4}{3}\pi r^3$

**Surface area of sphere** =  $4\pi r^2$



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Answer ALL TWENTY SIX questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1  $P = m^2 - 4c$

(a) Work out the value of  $P$  when  $m = -5$  and  $c = 3$

Plugging in:  $m = -5$  and  $c = 3$  into:  $P = m^2 - 4c$ :

$$P = (-5)^2 - 4(3)$$

$$P = 25 - 12$$

$$P = 13$$

$$P = 13 \quad (2)$$

(b) Expand and simplify  $(x+5)(x-7)$

using the box method:

	$x$	$5$	
$x$	$x^2$	$5x$	$= x^2 + 5x - 7x + 35$ collecting like terms $= x^2 - 2x + 35$
$-7$	$-7x$	$35$	

$$x^2 - 2x + 35 \quad (2)$$

(Total for Question 1 is 4 marks)

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- 2 Sandeep wants to buy some packets of pens and some boxes of pencils for his stationery shop.

Each packet of pens contains 9 pens.

Each box of pencils contains 12 pencils.

Each packet of pens costs £7.60

Each box of pencils costs £4.80

Sandeep can only buy full packets of pens and full boxes of pencils.

He wants to buy exactly the same number of pens as pencils.

Work out the minimum amount Sandeep needs to pay.

Find the lowest common multiple of 9 and 12.

Listing the multiples:

9, 18, 27, 36, 45

12, 24, 36

So the lowest common multiple of 9 and 12 is 36

So Sandeep will buy 36 pens and 36 pencils

To calculate how many boxes:

$$36 \div 9 = 4 \text{ boxes of pens}$$

$$36 \div 12 = 3 \text{ boxes of pencils}$$

Calculate cost:

$$4 \times 7.6 = 30.4$$

$$3 \times 4.8 = 14.4$$

$$\text{Total cost: } 30.4 + 14.4 = \text{£}44.80$$

£ 44.80

(Total for Question 2 is 4 marks)

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- 3 Anjali travels on the Eurostar train from Paris to Amsterdam.

The distance the train travels between Paris and Amsterdam is 515 km.

The time taken by the train to travel between Paris and Amsterdam is 3 hours 18 minutes.

Work out the average speed of the train.

Give your answer in km/h correct to the nearest whole number.

First, express 3 hrs 18 mins in just hrs.

60 mins in a hour, so 3 hrs 18 mins = 3 hrs +  $\frac{18}{60}$  hrs = 3.3 hrs



$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Speed} = \frac{515}{3.3}$$

$$\text{Speed} = 156.06$$

Rounded to the nearest whole number = 156 km/h

156 km/h

(Total for Question 3 is 3 marks)

- 4 Here are the first four terms of an arithmetic sequence.

38 31 24 17

Find an expression, in terms of  $n$ , for the  $n$ th term of the sequence.

$$a_n = a + (n-1)d$$

$a$  = value of term

$d$  = difference between terms

$$d = 31 - 38$$

$$d = -7$$

$$a_n = 38 + (n-1) - 7$$

$$a_n = 38 - 7n + 7$$

$$a_n = 45 - 7n$$

45 - 7n

(Total for Question 4 is 2 marks)



- 5 A field is in the shape of a trapezium.

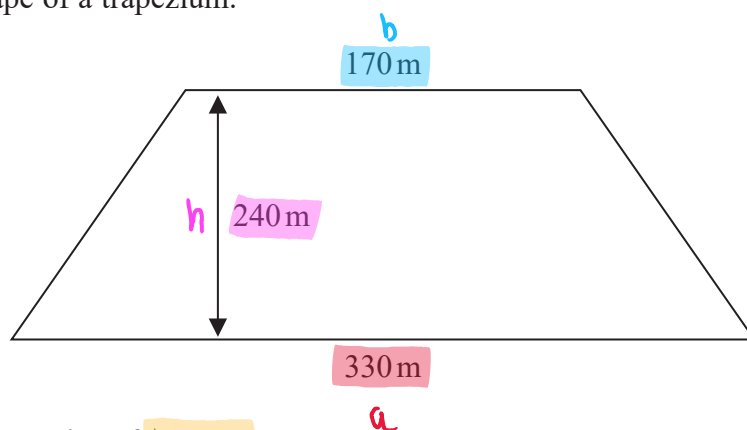


Diagram NOT accurately drawn

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The field is sold for a price of \$49 650

Given that 1 hectare = 10 000 m<sup>2</sup>

work out the average price of the field per hectare.

Calculate the area of the trapezium:

area of the trapezium:  $\frac{1}{2}(a+b) \times h$

$$= \frac{1}{2}(330 + 170) \times 240$$

$$= 60,000 \text{ m}^2$$

convert to hectares:

$$60,000 \div 10,000 = 6 \text{ ha}^2$$

$$\text{Average price: } 49650 \div 6 = \$8275$$

\$ 8275

(Total for Question 5 is 4 marks)



6 In his previous job, Pierre was paid 400 euros in total for working a 5-day week.

In his new job, Pierre is paid 14 euros per hour.

In his new job, Pierre works for 7 hours each day for a 5-day week.

(a) Work out the percentage increase in the amount that Pierre is paid for a 5-day week.

Calculate new weekly earnings:

€ per hr × hrs per day × 5 day week

$$14 \times 7 \times 5 = 490 \text{ euros per week}$$

Percentage change:  $\frac{\text{New} - \text{old}}{\text{old}} \times 100$

$$= \frac{490 - 400}{400} \times 100$$

$$= 22.5\% \text{ increase}$$

22.5%  
(4)

Marie changes her job.

Her salary decreases by 6%

Her new salary is 23 030 euros.

(b) Work out Marie's salary before she changes her job.

If her salary decreases by 6%, we can express the question as:

$$\begin{aligned} \frac{1 - 0.06}{100} & \rightarrow 94\% = 23\,030 \\ \frac{1}{100} & = 245 \quad \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} -94 \\ 100\% & = 24\,500 \quad \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} \times 100 \end{aligned}$$

Salary before she changed her job: 24 500 euros

24 500 euros  
(3)

(Total for Question 6 is 7 marks)



7 (a) Simplify  $(4^{-2})^0$

$$-2 \times 0 = 0$$

So equation becomes:  $4^0$

Anything to the power of 0 is 1

$$\text{So } 4^0 = 1$$

$$3^{-14} \times 3^8 = 3^m$$

(b) Find the value of  $m$

Using indices rules:  $x^a \times x^b = x^{a+b}$

$$\text{So, } 3^{-14} \times 3^8 = 3^{-14+8} = 3^{-6}$$

$$\therefore m = -6$$

$$m = -6 \quad (1)$$

(Total for Question 7 is 2 marks)

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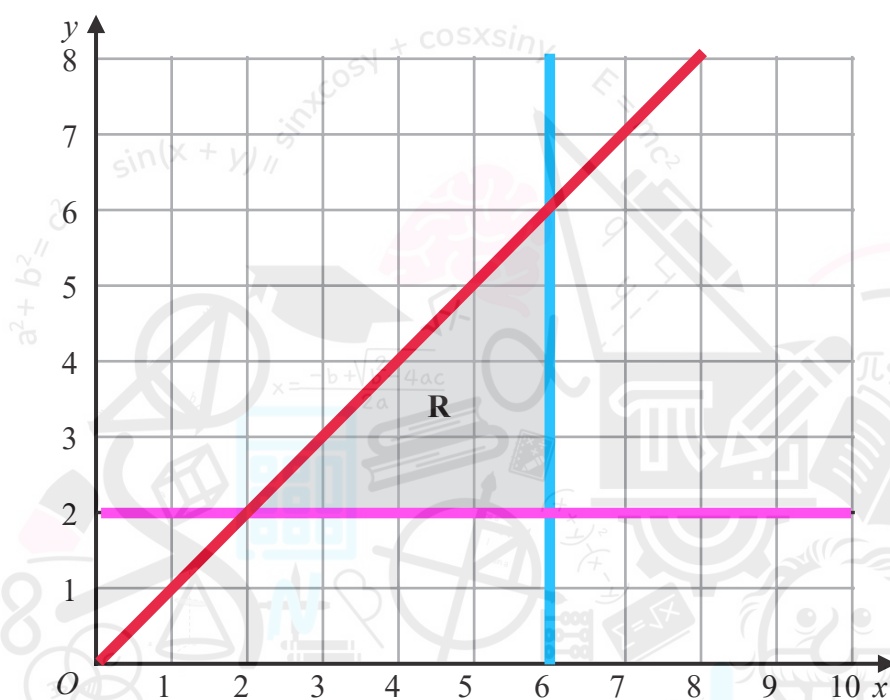


8 (a) Solve  $9 - 4x > 17$

$$\begin{aligned}
 9 - 4x &> 17 \\
 9 &> 17 + 4x && \text{+4x} \\
 4x + 17 &< 9 && \text{flip around} \\
 4x &< -8 \\
 x &< -2 && \text{-4}
 \end{aligned}$$

$$x < -2$$

(2)



(b) Write down the three inequalities that represent the shaded region **R**

The triangle falls to the left of the line, so we use  $\leq$ . Line is coming from the  $x$  value of 6, so inequality is  $x \leq 6$ .

Triangle falls above the line, so we use  $\geq$ . Line is coming from the  $y$  value of 2, so inequality is  $y \geq 2$ .

Triangle falls below the line, so we use  $\leq$ . Line is coming from the origin, so it links  $x$  and  $y$  together:  $y \leq x$ .

$$x \leq 6$$

$$y \geq 2$$

$$y \leq x$$

(3)

(Total for Question 8 is 5 marks)



9 The diagram shows a rectangular sheet of metal  $ABCD$

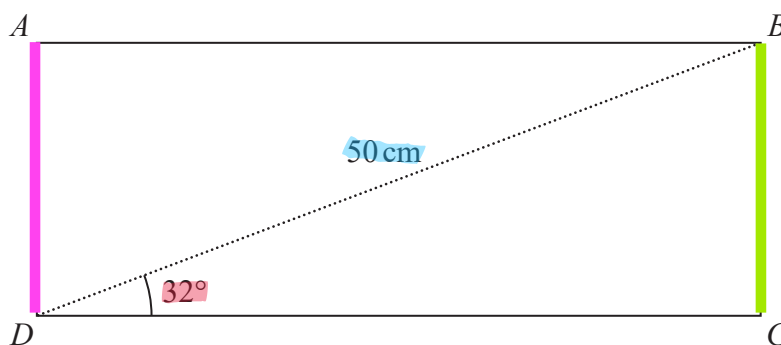


Diagram NOT accurately drawn

$BD = 50 \text{ cm}$  and angle  $BDC = 32^\circ$

Nasser joins side  $AD$  to side  $BC$  to form a cylinder.

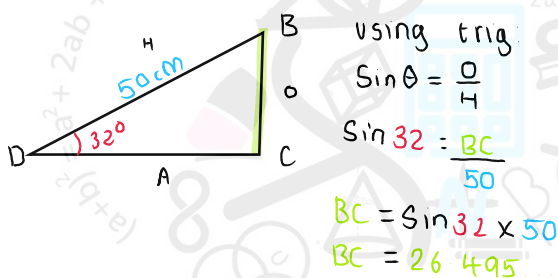
$BC$  is the height of the cylinder.

$DC$  is the circumference of the cross section of the cylinder.

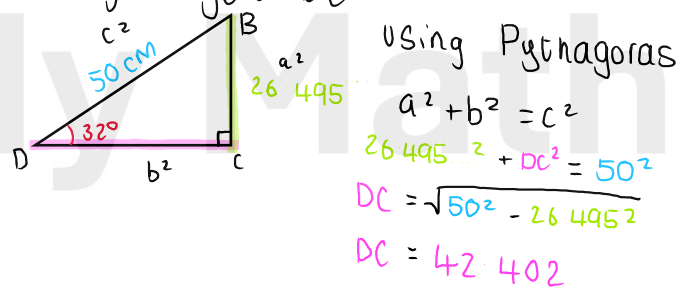
Work out the volume, in  $\text{cm}^3$ , of the cylinder.

Give your answer correct to 3 significant figures.

First calculate the length of  $BC$ :



finding length  $DC$ :



radius =  $\frac{\text{Circumference}}{2}$

radius =  $\frac{42.402\dots}{2\pi}$

radius = 6.74

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10 Gemara works as a taxi driver.

Last week, he recorded the following information about the distances he drove.

For the 5 days from Monday to Friday, the mean number of kilometres he drove was 104

For the 7 days from Monday to Sunday, the mean number of kilometres he drove was 127

On Saturday, Gemara drove 132 kilometres.

Work out the number of kilometres he drove on Sunday.

First calculate the total number of km across Mon-Fri and Mon-Sun.

Mon-Fri:

$$\text{mean number of km's} = \frac{\text{Total number of km's}}{\text{number of days}}$$

$$\text{Total number of km's} = \text{mean number of km's} \times \text{number of days}$$

$$\begin{aligned} \text{Total number of km's} &= 104 \times 5 \\ &= 520 \text{ km travelled Mon-Fri} \end{aligned}$$

Using the same formula as above for Mon-Sun:

$$\begin{aligned} \text{Total number of km's} &= 127 \times 7 \\ &= 889 \text{ km travelled Mon-Sun} \end{aligned}$$

Number of km's driven Sat-Sun:

$$889 - 520 = 369 \text{ km}$$

Number of km's driven on Sunday:

$$369 - 132 = 237 \text{ km}$$

..... 237 ..... kilometres

(Total for Question 10 is 3 marks)

11 Express  $\left(\frac{m^6 k^{10}}{25}\right)^{\frac{3}{2}}$  in the form  $\frac{m^a k^b}{c}$  where  $a$ ,  $b$  and  $c$  are integers to be found.

Breaking down the question term by term:

$$m^6 \times \frac{3}{2} = m^9$$

$$k^{10} \times \frac{3}{2} = k^{15}$$

$$25^{\frac{3}{2}} = 125$$

$$= \frac{m^9 k^{15}}{125}$$

$$\frac{m^9 k^{15}}{125}$$

(Total for Question 11 is 2 marks)

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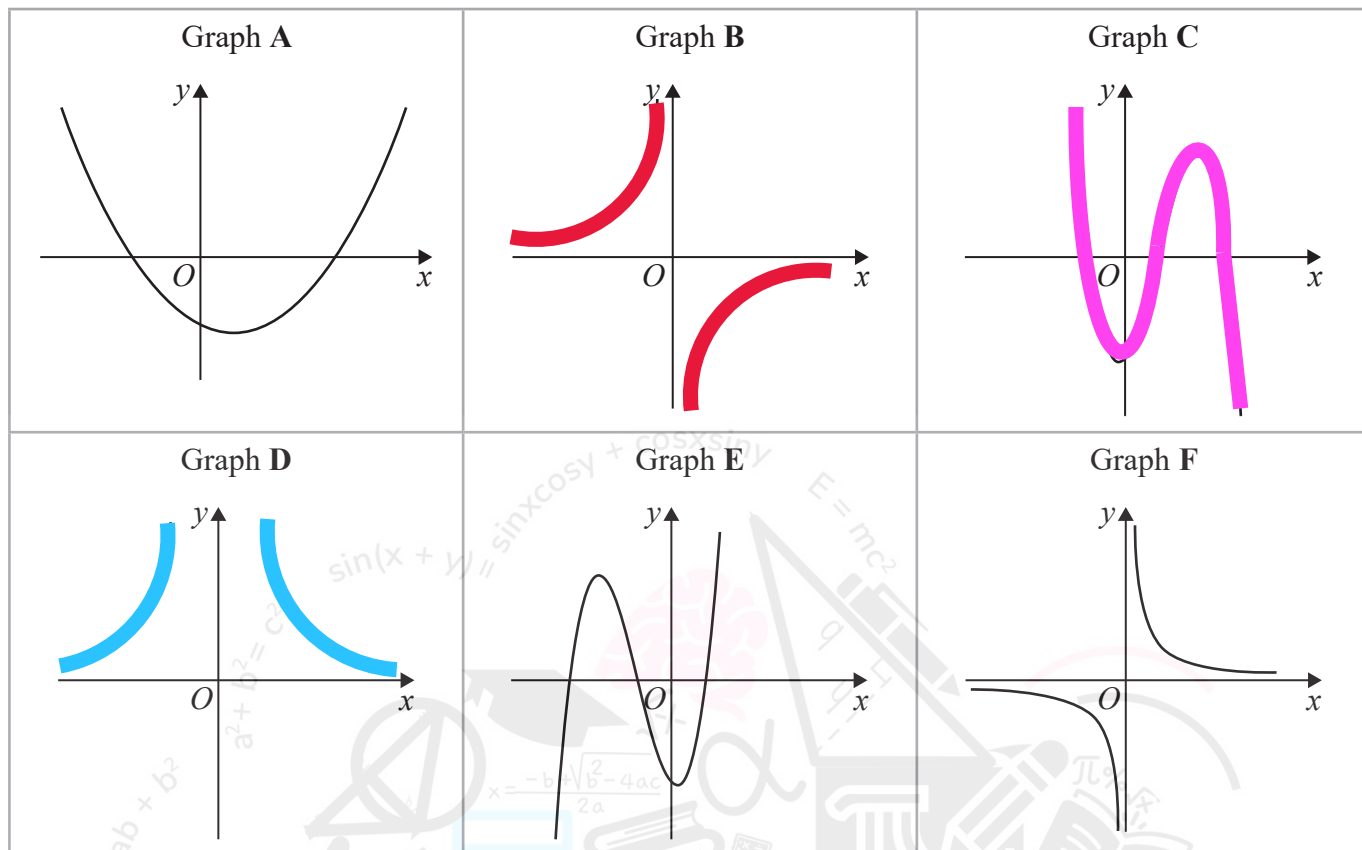


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12 Here are six graphs.



Write down the letter of the graph of

(a)  $y = \frac{10}{x^2}$

Graph D: recognise that  $\frac{1}{x^2}$  produces a  $\frac{1}{x^2}$  shaped graph, so the addition of 10 pulls the graph further away from the origin, producing graph D.

D  
.....  
(1)

(b)  $y = x - 3 + 3x^2 - x^3$

Graph C: recognise that the  $3x^2 - x^3$  establishes the  $\frac{1}{x^3}$  shape, the addition of  $x - 3$  'stretches' the graph more to produce graph C.

C  
.....  
(1)

(c)  $y = -\frac{3}{x}$

Graph B: recognise that the  $-\frac{1}{x}$  graph forms a  $\frac{1}{x}$  shape, so graph B shows this shape, with the addition of the graph being slightly further away from the origin to account for the 3.

B  
.....  
(1)

(Total for Question 12 is 3 marks)



13 Feruzi invests 80 000 Kenyan shillings (KES)

He invests the money for 3 years at  $x\%$  compound interest each year.

At the end of 3 years, the total interest he receives is 6151.25 KES

Work out the value of  $x$

If the total interest he receives is 6151.25 KES, then the total amount in his account after 3 years =  $80000 + 6151.25 = 86151.25$

Compound interest formula =  $\text{initial amount} \times \left(\frac{100 + \text{interest rate}}{100}\right)^{\text{number of years}}$

$$86151.25 = 80000 \times \left(\frac{100 + x}{100}\right)^3$$

$$86151.25 = 80000 \times (1 + x)^3$$

$$x = \sqrt[3]{\frac{86151.25}{80000}}$$

$$x = 1.025$$

$$x = 2.5\%$$

$$x = 2.5$$

(Total for Question 13 is 3 marks)

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14 Akari played a computer game **eleven times**.

Here are her scores.

25 20 28 27 26 22 23 29 20 29 26

(a) Find the interquartile range of her scores.

First, order the scores from lowest to highest:

20 20 22 23 25 26 26 27 28 29 29

$$IQR = UQ - LQ$$

$$UQ = 11 \times 0.75 = 8.25, \text{ so}$$

identify the 9<sup>th</sup> value = 28

$$LQ = 11 \times 0.25 = 2.75, \text{ so}$$

identify the 3<sup>rd</sup> value = 22

$$IQR = 28 - 22$$

$$IQR = 6$$

6  
(3)

Machi played the same computer game eleven times.

The interquartile range for Machi's scores was 9

(b) Who had the more consistent scores, Akari or Machi?

Give a reason for your answer.

Akari, as the IQR is lower/smaller.

(1)

(Total for Question 14 is 4 marks)



- 15 Osvaldo has a biased coin.  
He spins the coin three times.

The probability that the coin lands on a head three times is  $\frac{27}{64}$

Work out the probability that the coin will land on a tail three times.

$$\text{land on head 3 times} = \frac{27}{64}$$

$$\text{land on head once} = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

$$\text{Land on tails} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\text{land on tails 3 times} = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

(Total for Question 15 is 3 marks)

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16 Show that  $\frac{2\sqrt{3}}{\sqrt{3}-1}$  can be written in the form  $a + \sqrt{a}$  where  $a$  is an integer.

Show your working clearly.

Start by rationalising the denominator.

To do this, change the sign of  $\sqrt{3}-1$  to  $\sqrt{3}+1$ .

$$\frac{2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{3}$	$-1$
$1$	$2 \times 3 = 6$	$1$	$3 - \sqrt{3}$
$1$	$2\sqrt{3}$	$1$	$\sqrt{3} - 1$

$$= \frac{6+2\sqrt{3}}{2}$$

$$= 3 + \sqrt{3}$$

(Total for Question 16 is 3 marks)

17 Make  $x$  the subject of  $y = \sqrt[3]{\frac{6+5x}{x+4}}$

$$y = \sqrt[3]{\frac{6+5x}{x+4}}$$

cube each term to remove the cubed root

$$y^3 = \frac{6+5x}{x+4}$$

$$y^3(x+4) = 6+5x$$

$$xy^3 + 4y^3 = 6+5x$$

expand the brackets

$$xy^3 - 5x + 4y^3 = 6$$

take  $x$  terms to one side

$$xy^3 - 5x = 6 - 4y^3$$

$-4y^3$ , ready for factorising

$$x(y^3 - 5) = 6 - 4y^3$$

factorise  $x$

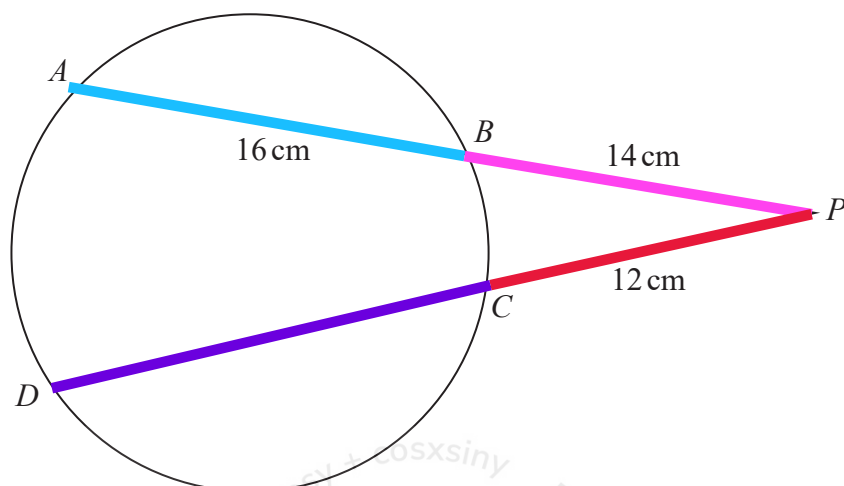
$$x = \frac{6 - 4y^3}{y^3 - 5}$$

$$x = \frac{6 - 4y^3}{y^3 - 5}$$

(Total for Question 17 is 4 marks)



18

Diagram NOT  
accurately drawn

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$A$ ,  $B$ ,  $C$  and  $D$  are points on a circle.

$ABP$  and  $DCP$  are straight lines.

$$AB = 16 \text{ cm}$$

$$BP = 14 \text{ cm}$$

$$CP = 12 \text{ cm}$$

Work out the length of  $DC$

$$DP \times CP = (AB + BP) \times BP$$

$$AB + BP = 16 + 14 = 30 \text{ (AP)}$$

$$DP \times CP = AP \times BP$$

$$DP \times 12 = 30 \times 14$$

$$DP \times 12 = 420$$

$$DP = \frac{420}{12}$$

$$DP = 35 \text{ cm}$$

$$DC = DP - CP$$

$$DC = 35 - 12$$

$$DC = 23 \text{ cm}$$

..... 23 ..... cm

(Total for Question 18 is 3 marks)



19 30 adults booked to stay in a hotel.

19 adults booked breakfast

15 adults booked dinner

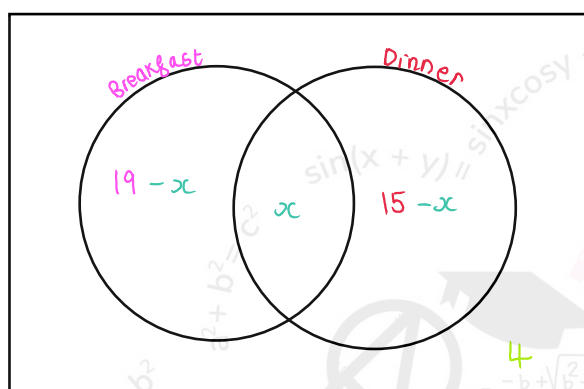
4 adults did not book breakfast or dinner

Some adults booked breakfast **and** dinner.

Meihui chooses at random two of the 30 adults.

Work out the probability that these two adults each booked breakfast **and** dinner.

Drawing a venn diagram can help us visualise the question:



We do not know how many adults booked breakfast and dinner. So let this =  $x$  in the Venn diagram

NOTE: To calculate how many adults ordered just breakfast, we subtract how many had breakfast and dinner from the total for breakfast (19, given to us in the question)

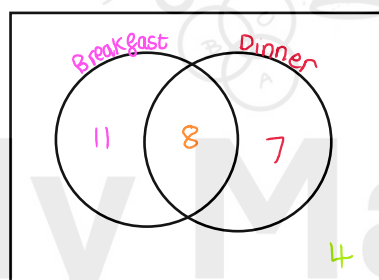
To find  $x$ :  $30 = 19 - x + x + 15 - x + 4$

$$30 = 38 - x$$

$$-8 = -x$$

$$x = 8$$

Venn diagram becomes:



Probability that both booked breakfast and dinner:

$$\frac{8}{30} \times \frac{8-1}{30-1}$$

$$= \frac{8}{30} \times \frac{7}{29}$$

$$= \frac{28}{435}$$

$$\frac{28}{435}$$

(Total for Question 19 is 4 marks)

20  $A$ ,  $B$  and  $C$  are points on a circle.

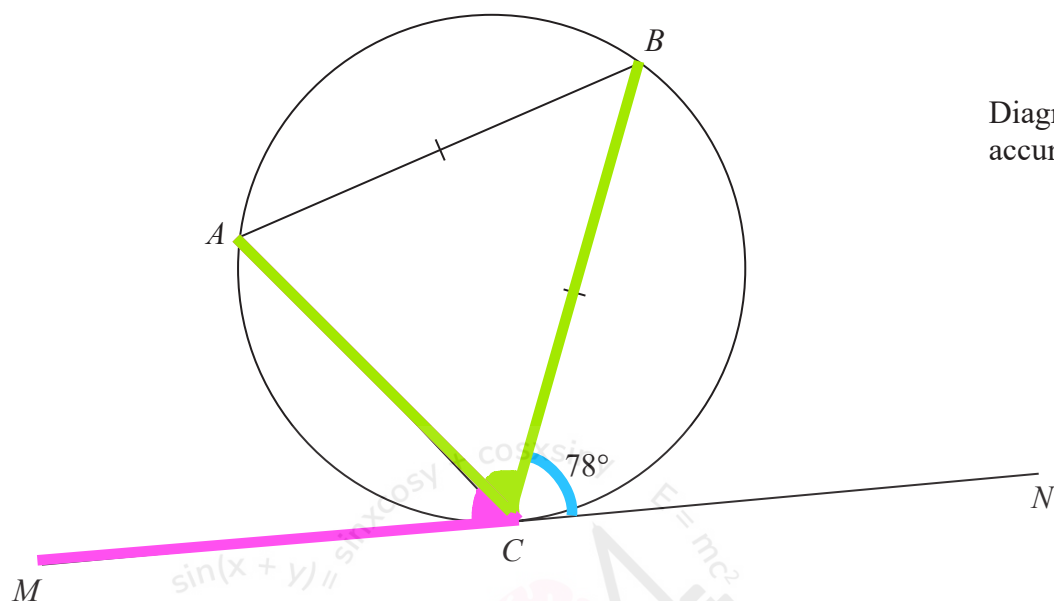


Diagram NOT  
accurately drawn

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$MN$  is the tangent to the circle at  $C$

$AB = CB$

Angle  $BCN = 78^\circ$

Find the size of angle  $ABC$

Using the rules that  $\hat{MCA} = \hat{BCN}$  and angles on a straight line =  $180^\circ$ :

$$\hat{ABC} = 180 - 78 - 78$$

$$\hat{ABC} = 24^\circ$$

24

(Total for Question 20 is 2 marks)

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21 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

$$= y = 1 + 2x \quad (2)$$

Show clear algebraic working.

plugging 1 into 2:

$$(1+2x)^2 + x(1+2x) = 7$$

expand the brackets

$$\begin{array}{r} 1 \quad 2x \\ 1 \quad 1 \quad 2x \\ 2x \quad 2x \quad 4x^2 \end{array}$$

$$4x^2 + 4x + 1 + x + 2x^2 = 7$$

collecting like terms

$$6x^2 + 5x + 1 = 7$$

$$6x^2 + 5x - 6 = 0$$

using the quadratic formula to solve for  $x$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+5 \pm \sqrt{5^2 - 4(6)(-6)}}{2(6)}$$

$$x = \frac{5 \pm 13}{12}$$

$$x = \frac{5+13}{12} \quad x = \frac{5-13}{12}$$

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

plugging back into  $y = 1 + 2x$ :

$$y = 1 + 2(-3/2) \quad y = 1 + 2(2/3)$$

$$y = -2 \quad y = \frac{7}{3}$$

$$= \left(-\frac{3}{2}, -2\right) \quad \text{and} \quad \left(\frac{2}{3}, \frac{7}{3}\right)$$

$$\left(-\frac{3}{2}, -2\right)$$

$$\left(\frac{2}{3}, \frac{7}{3}\right)$$

(Total for Question 21 is 5 marks)

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22 Here is a cuboid  $ABCDEFGH$

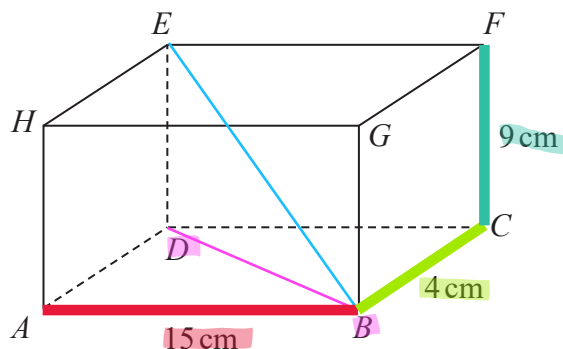


Diagram NOT accurately drawn

$$AB = 15 \text{ cm}$$

$$BC = 4 \text{ cm}$$

$$CF = 9 \text{ cm}$$

(a) Work out the length of  $BE$

Give your answer correct to 3 significant figures.

$$DB = AC$$

Using Pythagoras:

$$AC = \sqrt{15^2 + 4^2}$$

$$AC = \sqrt{241}$$

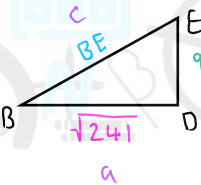
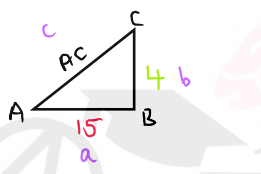
$$\therefore DB = \sqrt{241}$$

$$ED = FC, \therefore ED = 9$$

Using Pythagoras again:

$$BE = \sqrt{9^2 + (\sqrt{241})^2}$$

$$BE = 17.9 \text{ cm}$$



..... 17.9 cm  
(2)

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Here is a cuboid  $PQRSTUVW$

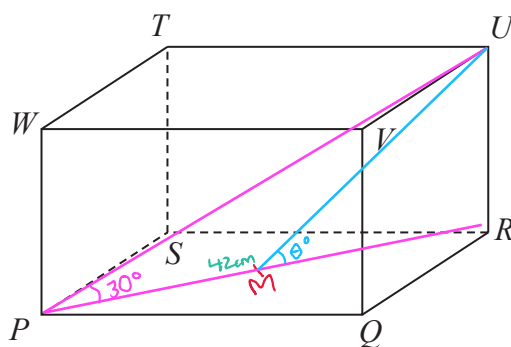


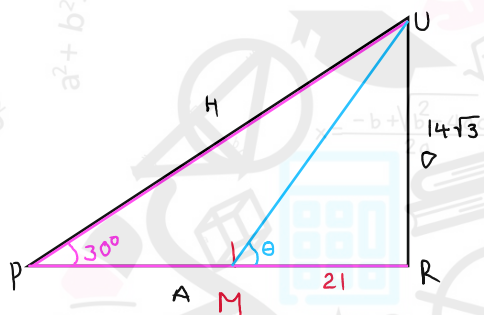
Diagram NOT  
accurately drawn

$$PR = 42 \text{ cm}$$

The size of the angle between  $PU$  and the plane  $PQRS$  is  $30^\circ$

$M$  is the midpoint of  $PR$

- (b) Work out the size of angle  $UMR$   
Give your answer correct to 3 significant figures.



$$MR = \frac{PR}{2}$$

$$MR = \frac{42}{2}$$

$$MR = 21$$

$$\tan \theta = \frac{UR}{MR}$$

$$\tan 30 = \frac{UR}{42}$$

$$UR = \tan 30 \times 42$$

$$UR = 14\sqrt{3}$$

$$\tan \theta = \frac{14\sqrt{3}}{21}$$

$$\theta = \tan^{-1} \left( \frac{14\sqrt{3}}{21} \right)$$

$$\theta = 49.1^\circ$$

$$\underline{\quad 49.1^\circ \quad}$$

(3)

(Total for Question 22 is 5 marks)



23 Here are the first three terms of an arithmetic sequence.

$$8p \quad 7p-3 \quad 4p+2$$

The sum of the first  $n$  terms of the sequence is  $-1914$

Work out the value of  $n$

Show your working clearly.

First calculate the value of  $p$ :

The differences between the terms are equal, so:

$$(7p-3) - (8p) = (4p+2) - (7p-3)$$

$$-p - 3 = -3p + 5$$

$$p = 4$$

terms become:

$$\begin{array}{ccc} 8(4) & 7(4)-3 & 4(4)+2 \\ 32 & 25 & 18 \end{array}$$

sum of series:

$$S_n = \frac{n}{2} [2a + (n-1)(r)]$$

$$f = 25 - 32$$

$$r = -7$$

$$a = 32$$

$$S_n = \frac{n}{2} [2(32) + (n-1)(-7)] = -1914$$

expanding gives:

$$7n^2 - 71n - 3828 = 0$$

using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{71 \pm \sqrt{(-71)^2 - 4(7)(-3828)}}{2(7)}$$

$$x = 29 \quad \text{or} \quad x = \frac{-182}{7}$$

$x$  cannot be negative,  $\therefore x = 29$

$$n = 29$$

(Total for Question 23 is 5 marks)

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





- 24 The surface area of sphere A is nine times the surface area of sphere B  
The difference between the volume of sphere A and the volume of sphere B is  $117\pi \text{ cm}^3$

Find the radius of the smaller sphere.  
Show your working clearly.

$$\text{Surface area scale factor} = 9$$

$$\text{Let surface area} = x^2$$

$$\text{Volume} = x^3$$

$$\text{Length} = x$$

$$x^2 = 9$$

$$x = 3$$

$$x^3 = 12$$

$$\text{volume scale factor} = 12$$

$$\text{length scale factor} = 3$$

$$\text{volume of sphere} = \frac{4}{3}\pi r^2$$

$$\text{volume of B} - \text{volume of A} = 117\pi$$

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

rearranging gives:

$$r^3 = \frac{117 \times 3}{104}$$

$$r = \sqrt[3]{\frac{117 \times 3}{104}}$$

$$r = \frac{3}{2}$$

$$\dots\dots\dots \frac{3}{2} \dots\dots\dots \text{cm}$$

(Total for Question 24 is 5 marks)



- 25 The straight line with equation  $y - 2x = 7$  is the perpendicular bisector of the line  $AB$  where  $A$  is the point with coordinates  $(j, 7)$  and  $B$  is the point with coordinates  $(6, k)$

Find the coordinates of the midpoint of the line  $AB$

Show clear algebraic working.

for the gradients:

$$m_1 \times m_2 = -1$$

$$m_1 = \text{gradient of } y = 2x + 7 = 2$$

$$m_2 = \text{gradient of } AB$$

$$2 \times m_2 = -1$$

$$m_2 = -\frac{1}{2}$$

$$\text{gradient}_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k - 7}{6 - j}$$

$$-\frac{1}{2} = \frac{k - 7}{6 - j}$$

$$-\frac{1}{2}(6 - j) = k - 7$$

$$-6 + j = 2k - 14$$

$$2k - 8 = j \quad (1)$$

$$\text{midpoint } AB = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{j + 6}{2}, \frac{7 + k}{2}$$

plugging into  $y = 2x + 7$

$$\frac{7 + k}{2} = 2\left(\frac{j + 6}{2}\right) + 7$$

$$= 7 + k = 2j + 12 + 14$$

$$k = 2j + 19 \quad (2)$$

plugging 2 into 1

$$2(2j + 19) - 8 = j$$

$$4j + 38 - 8 = j$$

$$3j = -30$$

$$j = -10$$

$$k = 2j + 19$$

$$k = 2(-10) + 19$$

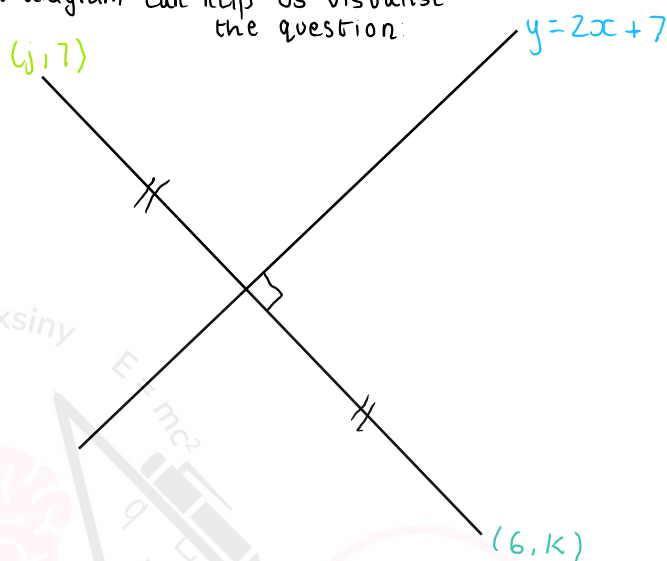
$$k = -1$$

$$\text{plugging into } \left(\frac{j + 6}{2}, \frac{7 + k}{2}\right)$$

$$\left(\frac{-10 + 6}{2}, \frac{7 - 1}{2}\right)$$

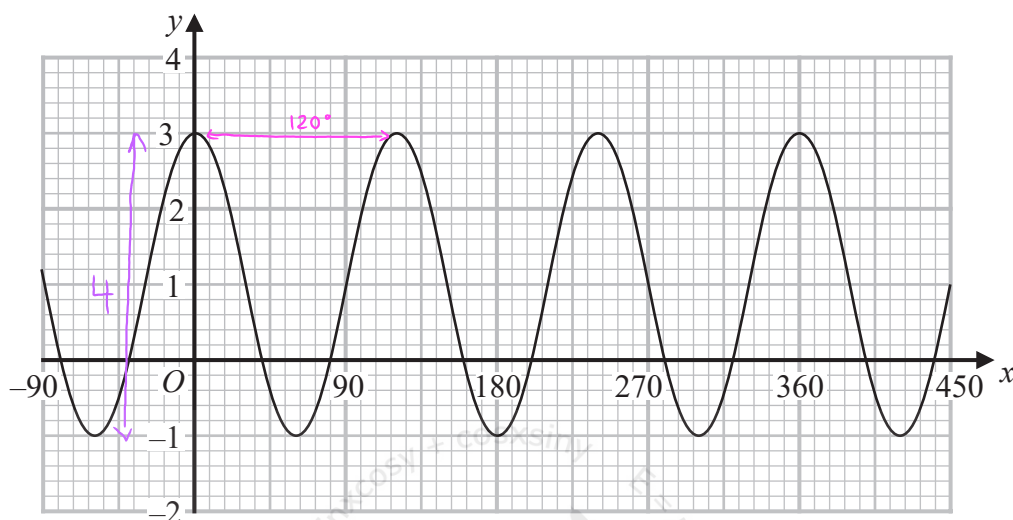
$$= (-2, 3)$$

Drawing a diagram can help us visualise the question:





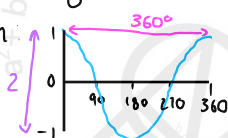
26 Here is a sketch of the curve with equation  $y = a \cos bx^\circ + c$  where  $-90 \leq x \leq 450$



Find the value of  $a$ , the value of  $b$  and the value of  $c$

This is a transformation of the cos graph

Typical cos graph:



Finding  $b$

one period (where the line returns back to 1) is  $360^\circ$  on the typical cos graph. Applying this to the transformed graph gives  $120^\circ$

$$\text{So } b = \frac{360}{120}$$

$$b = 3$$

$$a = 2$$

$$b = 3$$

$$c = 1$$

(Total for Question 26 is 3 marks)

TOTAL FOR PAPER IS 100 MARKS

Finding  $a$ :

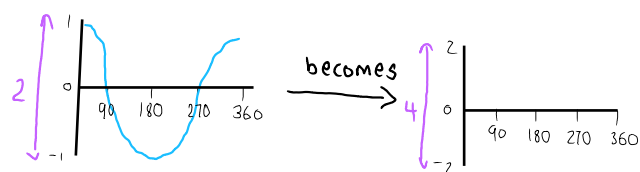
Now we compare how much the graphs have been stretched vertically:

$$\text{So } a = \frac{4}{2}$$

$$a = 2$$

Finding  $c$

Apply the scale factor of 2 to the typical cos graph.



Comparing the new vertical height:

$$c = \frac{4}{4}$$

$$c = 1$$

